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Interactions of cosmic neutrinos with nucleons in the RS model

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Abstract. We consider the scattering of brane fields due to t-channel massive graviton exchanges in the Randall–Sundrum model. The eikonal amplitude is analytically calculated and both differential and total neutrino–nucleon cross sections are estimated. The event rate of quasi-horizontal air showers induced by cosmic neutrinos, which can be detected at the Pierre Auger Observatory, is presented for two different fluxes of cosmogenic neutrinos.

1 Introduction

The detection of air showers induced by ultra-high energy neutrinos may help to solve many important problems, such as the propagation of cosmic neutrinos to the Earth and their interactions with the nucleons at energies around tens (hundreds) of TeV. In this energy region, neutrino–nucleon interactions may be strong due to new physics. There is a large class of models in a space-time with extra spacial dimensions which result in a new TeV phenomenology. In the present paper we will consider an approach with non-factorizable metric proposed in [1,2] and study scattering of the SM fields in this scenario.

The RS model [1,2] is a model of gravity in a slice of a 5-dimensional anti-de Sitter space (AdS_5) with a single extra dimension compactified to the orbifold S^1/Z_2. The metric is of the form

$$\mathrm{d}s^2 = \mathrm{e}^{-2\kappa|y|}\,\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + \mathrm{d}y^2. \qquad (1)$$

Here $y = r_\mathrm{c}\theta$ ($0 \leqslant \theta \leqslant \pi$), r_c is the "radius" of the extra dimension, and the parameter κ defines a scalar (negative) curvature of the space.

From a 4-dimensional action one can derive the relation

$$\bar{M}_\mathrm{Pl}^2 = \frac{M^3}{\kappa}\left(1 - \mathrm{e}^{-2\pi\kappa r}\right) \simeq \frac{M^3}{\kappa}, \qquad (2)$$

which means that $\kappa \sim \bar{M}_\mathrm{Pl} \sim M$, with M being a 5-dimensional Planck scale.

We will consider the so-called RS1 model [1] which has two 3-dimensional branes with equal but opposite sign tensions which are located at the point $y = \pi r_\mathrm{c}$ (called the TeV brane) and at the point $y = 0$ (referred to as the Planck brane). All SM fields are constrained to the TeV brane, while the gravity propagates in the bulk (all spatial dimensions).

From the point of view of an observer located on the TeV brane, there exist an infinite number of graviton Kaluza–Klein (KK) excitations with masses

$$m_n = x_n \kappa\, \mathrm{e}^{-\pi\kappa r_\mathrm{c}}, \qquad n = 1, 2, \ldots, \qquad (3)$$

where x_n are zeros of the Bessel function $J_1(x)$:[1]

$$J_1(x_n) = 0, \qquad n = 1, 2, \ldots \qquad (4)$$

By using a linear expansion of the metric, one can derive the interaction Lagrangian[2]

$$\mathcal{L} = -\frac{1}{\bar{M}_\mathrm{Pl}}\,T_{\mu\nu}\,h^{(0)}_{\mu\nu} - \frac{1}{\Lambda_\pi}\,T_{\mu\nu}\sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}, \qquad (5)$$

where

$$\Lambda_\pi = \bar{M}_\mathrm{Pl}\,\mathrm{e}^{-\pi\kappa r_\mathrm{c}} \qquad (6)$$

is the physical scale on the TeV brane. It can be chosen as small as 1 TeV for a thick slice of the AdS_5, $r_\mathrm{c} \simeq 12/\kappa \simeq 60\, l_\mathrm{Pl}$. We see from (5) that the couplings of all massive states are suppressed by Λ_π^{-1} only, while the zero mode couples with usual strength defined by the reduced Planck mass $\bar{M}_\mathrm{Pl} = M_\mathrm{Pl}/\sqrt{8\pi}$.

The main phenomenological parameters of the model are the scale Λ_π and the ratio

$$\mu = \frac{\kappa}{\bar{M}_\mathrm{Pl}}. \qquad (7)$$

The present experimental data together with theoretical bounds on the curvature of the AdS_5 restrict the allowed region for the variable μ (see, for instance, Fig. 1, taken from [3]):

$$0.01 \lesssim \mu \lesssim 0.1. \qquad (8)$$

[1] The first four values of x_n are 3.83, 7.02, 10.17, and 13.32.

[2] We do not consider a radion field [6] here because it is irrelevant for high-energy t-channel exchanges.

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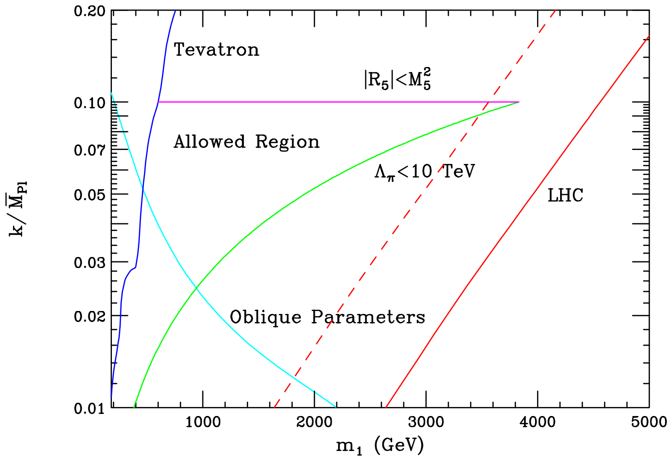

Fig. 1. Experimental and theoretical constraints on the RS model in the two-parameter $\kappa/\bar{M}_{\rm Pl}$–m_1 plane [3]. The allowed region lies in the center as indicated

The allowed value of μ is restricted by the so-called naturalness and requiring the 5-dimensional curvature to be small enough to consider linearized gravity on the brane. Thus, the lightest masses of the KK graviton modes, $m_n = x_n \mu \Lambda_\pi$, are of the order of 1 TeV [4, 5].

Our paper has the following structure. In the next section we consider interactions of the SM fields on the brane in the RS model induced by exchanges of massive gravitons. The eikonal amplitude is calculated and the elastic cross section for different sets of the RS parameters and/or invariant energy is estimated. In Sect. 3 we use these results to study the scattering of ultra-high energy cosmic neutrino off atmospheric nucleons. The neutrino–nucleon cross section is calculated and the event rate of quasi-horizontal neutrino showers expected at the Auger Observatory is presented. Our conclusions and discussions are the topics of Sect. 4.

2 Eikonal amplitude in the RS model

In what follows, we will employ the zero width approximation for the graviton KK resonances. The Born amplitude corresponding to t-channel exchange looks like (both the massless mode and KK gravitons contribute)

$$A_{\rm B}(s,t) = \frac{8\pi G_{\rm N} s^2}{-t} + \frac{s^2}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{-t + m_n^2}. \tag{9}$$

The sum in (9) converges very rapidly in n, since $x_n = \pi\,(n+1/4) + O(n^{-1})$ [7]. We consider the scattering of two particles *living on the TeV brane*. Thus, in (9) t means the 4-dimensional momentum transfer which is well-defined and conserved.

Let us underline that in (9) we sum spin-two particles with different KK numbers n (non-reggeized KK gravitons). In a more general approach, one should sum Regge trajectories $\alpha_n(t)$ which are numerated by n (KK charged gravireggeons). For the ADD model, this was done in [8]. The results of [9,10] can be reproduced in the limit $\alpha_n(t) \to 2$. As for the RS model, results on the gravireggeon contribution to the eikonal amplitude are presented in [13].

Generally, the massive KK states may decay to a pair of SM particles. The partial widths are proportional to m_n^3/Λ_π^2, where m_n is the mass of the resonance. In particular, the partial decay widths to massless gauge bosons, fermions, and a Higgs pair are[3]

$$\Gamma(h^{(n)} \to VV) = N_V\, a\, \frac{m_n^3}{40\pi\Lambda_\pi^2},$$
$$\Gamma(h^{(n)} \to f\bar{f}) = N_f\, \frac{m_n^3}{160\pi\Lambda_\pi^2},$$
$$\Gamma(h^{(n)} \to H\bar{H}) = \frac{m_n^3}{480\pi\Lambda_\pi^2}. \tag{10}$$

Here $N_V = 1(8)$ for photons and electroweak bosons (gluons), $N_f = 1(3)$ for the lepton (quark pair) mode, and $a = 1/2$ for identical particles. Then for the total width of the massive KK graviton in the RS model, Γ_n, we get the estimate (see also [12])

$$\Gamma_n \simeq m_n\,(0.31\,\mu\, x_n)^2. \tag{11}$$

Since the sum which we are interested in converges very rapidly in n (see the comment after (9)), we conclude from (11) and (8) that effectively $\Gamma_n/2 \ll m_n$.

The sum (9) can be calculated analytically by the use of the formula [7]

$$\sum_{n=1}^{\infty} \frac{1}{z_{n,\nu}^2 - z^2} = \frac{J_{\nu+1}(z)}{2\,z\,J_\nu(z)}, \tag{12}$$

where $z_{n,\nu}$ $(n = 1,2,\ldots)$ are zeros of the function $z^{-\nu} J_\nu(z)$. As a result, we obtain

$$A_{\rm B}(s,t) = \frac{8\pi G_{\rm N} s^2}{-t} + \frac{s^2}{2\mu\Lambda_\pi^3} \frac{1}{\sqrt{-t}} \frac{I_2(v)}{I_1(v)}. \tag{13}$$

Here $I_i(z)$ $(i = 1,2)$ are the modified Bessel functions and $v = \sqrt{-t}/\mu\Lambda_\pi$. Taking into account the properties of $I_i(x)$, we conclude from (13) that the contribution of the massive graviton modes dominates at large $|t|$:

$$A_{\rm B}(s,t)|_{|t|\gg\mu\Lambda_\pi} \simeq \frac{s^2}{2\mu\Lambda_\pi^3} \frac{1}{\sqrt{|t|}}. \tag{14}$$

Note that we would get another asymptotics in t, namely, $A_{\rm B}(s,t) \sim |t|^{-1}$, if we sum only a finite number of massive gravitons.

As it was shown in [9], it is ladder diagrams that make a leading contribution of the KK gravitons to the amplitude and results in the eikonal representation for the amplitude ($q^2 = -t$):

$$A^{\rm eik}(s,t) = 2{\rm i}s \int {\rm d}^2 b\, {\rm e}^{{\rm i}q\,b} \left[1 - {\rm e}^{{\rm i}\chi(s,b)}\right], \tag{15}$$

[3] These expressions can be obtained by the replacement $\bar{M}_{\rm Pl}^{-2} \to \Lambda_\pi^{-2}$ in the corresponding formulae derived for large extra dimensions in [11]. We have also neglected the masses of the SM particles, since $m_{\rm SM} \ll m_n$.

with the eikonal given by

$$\chi(s,b) = \frac{1}{4\pi s} \int \mathrm{d}q\, q\, J_0(q\,b)\, A_{\mathrm{B}}(s,-q^2). \tag{16}$$

The proper accounting for the massless mode has been presented in [10]. The result is

$$A^{\mathrm{eik}}(s,t) = \mathrm{e}^{\mathrm{i}\phi_4} \left\{ \frac{8\pi G_{\mathrm{N}} s}{-t} \frac{\Gamma(1-\mathrm{i}G_{\mathrm{N}}s)}{\Gamma(1+\mathrm{i}G_{\mathrm{N}}s)} + 4\pi \mathrm{i} s\,(-t)^{-\mathrm{i}G_{\mathrm{N}}s} \int_0^\infty \mathrm{d}b\, b^{1-2\mathrm{i}G_{\mathrm{N}}s} J_0\left(b\sqrt{|t|}\right) \times \left[1-\mathrm{e}^{\mathrm{i}\chi_{\mathrm{mass}}(s,b)}\right] \right\}. \tag{17}$$

Here $\chi_{\mathrm{mass}}(s,b)$ denotes the contribution of the massive modes to the eikonal, G_{N} is the Newton constant, and ϕ_4 is a 4-dimensional (infinite) phase. The first term in the RHS of (17) is the well-known 4-dimensional result derived by different methods in [14]. It is negligible at any conceivable energy and momentum transfer, and we can write (up to a phase factor)

$$A^{\mathrm{eik}}(s,t) \simeq 4\pi \mathrm{i} s \int_0^\infty \mathrm{d}b\, b\, J_0\left(b\sqrt{|t|}\right) \left[1-\mathrm{e}^{\mathrm{i}\chi_{\mathrm{mass}}(s,b)}\right]. \tag{18}$$

It follows from (13) that the eikonal depends on two dimensionless variables, s/Λ_π^2 and

$$u = b\mu\Lambda_\pi, \tag{19}$$

and it looks like

$$\tilde{\chi}_{\mathrm{mass}}(s,u) \equiv \chi_{\mathrm{mass}}\left(s, \frac{u}{\mu\Lambda_\pi}\right) = \frac{1}{8\pi} \frac{s}{\Lambda_\pi^2} \int_0^\infty \mathrm{d}v\, J_0(uv) \frac{I_2(v)}{I_1(v)}. \tag{20}$$

The eikonal (20) is very well approximated by the following expression (see the appendix for details):

$$\tilde{\chi}_{\mathrm{mass}}(s,u) \simeq \frac{\sqrt{3}}{16\pi} \frac{s}{u\Lambda_\pi^2} \exp(-2\sqrt{3}\,u). \tag{21}$$

At $\sqrt{s} \gg 5\,\Lambda_\pi$, the eikonal is exponentially small outside the region

$$b \lesssim b_0(s) = \frac{1}{\sqrt{3}\,\mu\Lambda_\pi} \ln \frac{\sqrt{s}}{\Lambda_\pi}. \tag{22}$$

At $b \to 0$, it is proportional to b^{-1}. Thus, we can roughly estimate the high-energy behavior of the elastic cross section:

$$\sigma_{\mathrm{el}}(s) \simeq \frac{\pi}{3\,(\mu\Lambda_\pi)^2} \ln^2 \frac{\sqrt{s}}{\Lambda_\pi} \approx \frac{\pi}{m_1^2} \ln^2 \frac{s}{\Lambda_\pi^2}, \tag{23}$$

where m_1 is the mass of the lightest KK graviton.

Let us underline that the Froissart–Martin-like formula (23) describes the contribution of the massive graviton modes. The presence of the *massless* graviton in the theory should result in *infinite* elastic and total cross sections [15]. However, its contribution can be safely neglected in our further calculations.

We can rewrite (18) in the form

$$A^{\mathrm{eik}}(s,t) \simeq 4\pi \mathrm{i} \frac{s}{(\mu\Lambda_\pi)^2} \int_0^\infty \mathrm{d}u\, u\, J_0\left(u\frac{\sqrt{-t}}{\mu\Lambda_\pi}\right) \left[1-\mathrm{e}^{\mathrm{i}\tilde{\chi}_{\mathrm{mass}}(s,u)}\right]. \tag{24}$$

Correspondingly, the differential cross section in the dimensionless variable

$$y = \frac{-t}{s} \tag{25}$$

is defined by

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}y} = \frac{1}{16\pi s} |A^{\mathrm{eik}}(s,-ys)|^2, \tag{26}$$

and we get the estimate

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}y}\right|_{y=0} \simeq \frac{\pi\, s}{36\,(\mu\Lambda_\pi)^4} \ln^4 \frac{\sqrt{s}}{\Lambda_\pi}. \tag{27}$$

It follows from (26) and (24) that $\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}y$ depends only on the variable y of (25), the parameter μ, and the ratio $\sqrt{s}/\Lambda_\pi$, in addition to the dimensional factor $(\mu\Lambda_\pi)^{-2}$, which defines the magnitude of the cross section. In particular, we have $\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}y|_{y=0} = s\,(\mu\Lambda_\pi)^{-4} f(s/\Lambda_\pi^2)$, and $\sigma_{\mathrm{el}} = (\mu\Lambda_\pi)^{-2} g(s/\Lambda_\pi^2, \mu)$, were $f(x)$ and $g(x,y)$ are dimensionless functions defined via the eikonal.

The results of our calculations with the use of formulae (24) (26) and (20) are presented in Figs. 2–5. The curves in Fig. 2 which show the energy dependence of the cross section were obtained for $\Lambda_\pi = 2$ TeV and $\mu = 0.05$. The

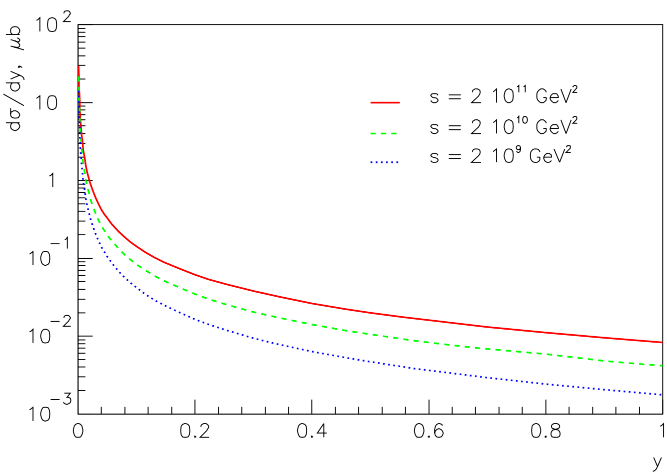

Fig. 2. The differential cross section as a function of dimensionless variable y for three fixed values of the invariant energy. The parameters of the RS model are chosen to be $\Lambda_\pi = 2$ TeV, and $\mu = 0.05$

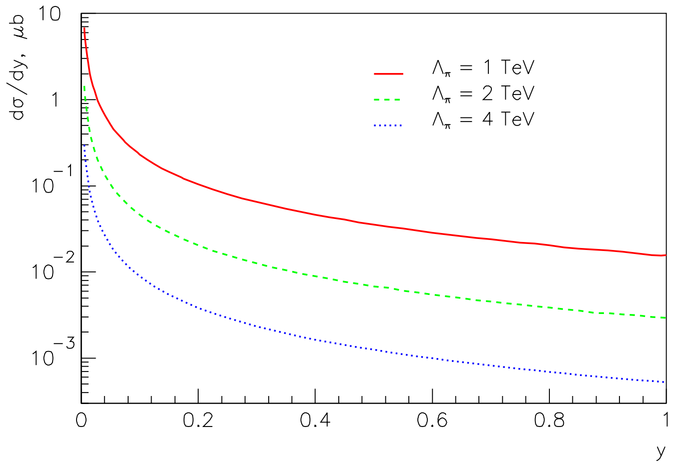

Fig. 3. The differential cross section as a function of variable y for three values of the mass scale Λ_π at fixed energy $s = 2 \cdot 10^{11}\,\mathrm{GeV}^2$ (with the parameter $\mu = 0.1$)

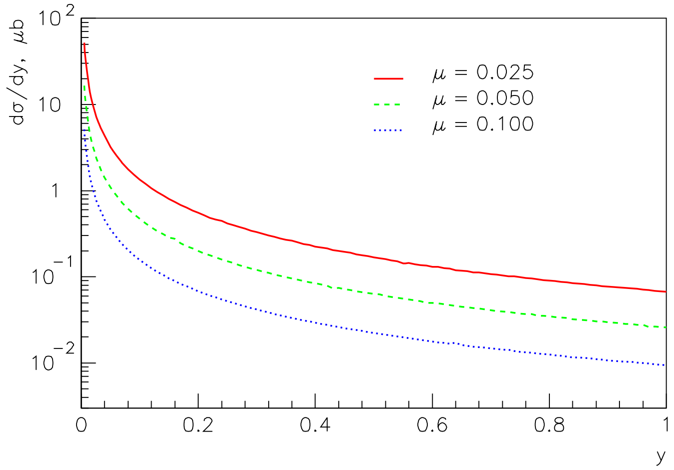

Fig. 4. The differential cross section as a function of variable y for three values of the RS parameter μ at fixed energy $s = 2 \cdot 10^{10}\,\mathrm{GeV}^2$ (with the scale $\Lambda_\pi = 1\,\mathrm{TeV}$)

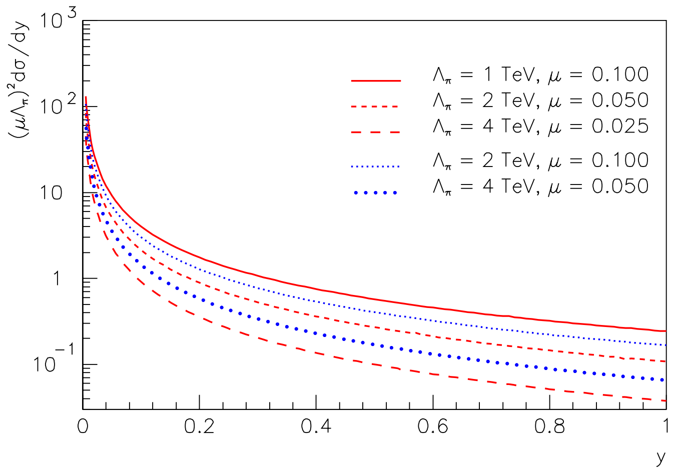

Fig. 5. The reduced (dimensionless) differential cross section as a function of variable y for different parameter sets (Λ_π, μ) at invariant energy $s = 2 \cdot 10^{10}\,\mathrm{GeV}^2$. The product $\mu\Lambda_\pi$ is taken to be 100 GeV (200 GeV) for the three first (two last) sets

dependence of the differential cross section on the parameter Λ_π at $s = 2 \cdot 10^{11}\,\mathrm{GeV}^2$, $\mu = 0.1$, is presented in Fig. 3. Next Fig. 4 demonstrates the dependence of $\mathrm{d}\sigma/\mathrm{d}y$ on the parameter μ at $s = 2 \cdot 10^{10}\,\mathrm{GeV}^2$, $\Lambda_\pi = 1\,\mathrm{TeV}$. Finally, in Fig. 5 the reduced differential cross section (namely, the cross section multiplied by the factor $(\mu\Lambda_\pi)^2$) is shown for several sets (Λ_π, μ).

3 Neutrino–nucleon cross section and neutrino-induced air showers

Let us now estimate the neutrino–nucleon differential cross section as a function of the variable y. The neutrino scatters off quarks and gluons which are distributed inside the nucleon. Thus, the neutrino–nucleon cross section is represented by

$$\frac{\mathrm{d}\sigma^{\nu N}(s)}{\mathrm{d}y} = \int\limits_{x_{\min}}^{1} \mathrm{d}x \sum_i f_i(x, M^2)\, \frac{\mathrm{d}\sigma^{\nu i}(\hat{s})}{\mathrm{d}y}, \tag{28}$$

where $f_i(x, M^2)$ is the distribution of parton i in momentum fraction x, and $\hat{s} = xs$ is the invariant energy of the partonic subprocess. The partonic differential cross section, $\mathrm{d}\sigma^{\nu i}(\hat{s})/\mathrm{d}y$, is defined via the eikonal (21) taken at the energy $\sqrt{\hat{s}}$.

We use the set of parton distribution functions (PDFs) from [16] based on the analysis of existing deep inelastic data in the next-to-leading order QCD approximation in the fixed-flavor-number scheme. The extraction of the PDFs is performed in [16] simultaneously with the value of the strong coupling and higher-twist contributions to the structure functions. The PDFs are available in the region $10^{-7} < x < 1$, $2.5\,\mathrm{GeV}^2 < Q^2 < 5.6 \cdot 10^7\,\mathrm{GeV}^2$ [16]. So no extrapolation in the variable x is needed.

We put $x_{\min} = \Lambda_\pi^2/s$ in (28). Since the eikonal is effectively cut at $b = b_0(\hat{s})$, see (22), we take the mass scale in the PDFs to be $M = 1/b_0(\sqrt{\hat{s}})$. The effective impact parameter b_0 is much smaller than the size of the nucleon. Thus, our assumption that the neutrino interacts with the constituents of the nucleon and probes its inner structure is well justified.

The differential cross section as a function of y, the energy fraction deposited from the neutrino to the nucleon, is presented in Fig. 6 for the neutrino energy $E_\nu = 10\,\mathrm{EeV}$ and three sets of parameters of the RS model.

In order to estimate the effective range of the variable y which contributes to the neutrino–nucleon cross section, we have calculated the quantity

$$\sigma^{\nu N}(y > y_0) = \int\limits_{y_0}^{1} \frac{\mathrm{d}\sigma^{\nu N}}{\mathrm{d}y}, \tag{29}$$

where y_0 is the minimum fraction of energy lost by the neutrino (deposited to the nucleon). The dependence of the quantity $\sigma_{\mathrm{el}}(y > y_0)$ on y_0 at different values of the neutrino energy E_ν is shown in Fig. 7 for $\Lambda_\pi = 2\,\mathrm{TeV}$,

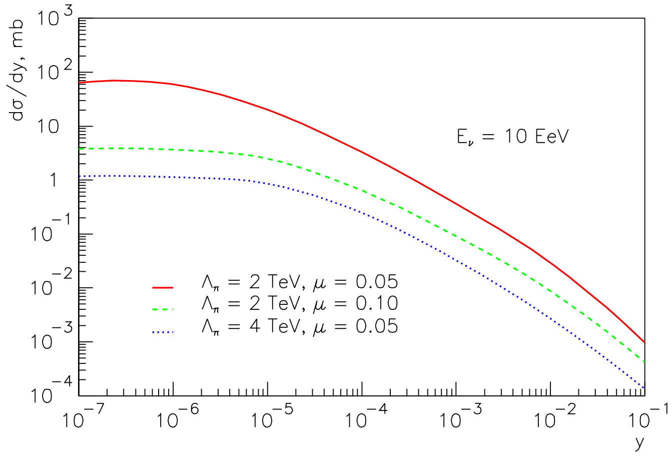

Fig. 6. The differential neutrino–proton cross section as a function of y, the fraction of the neutrino energy deposited to the proton

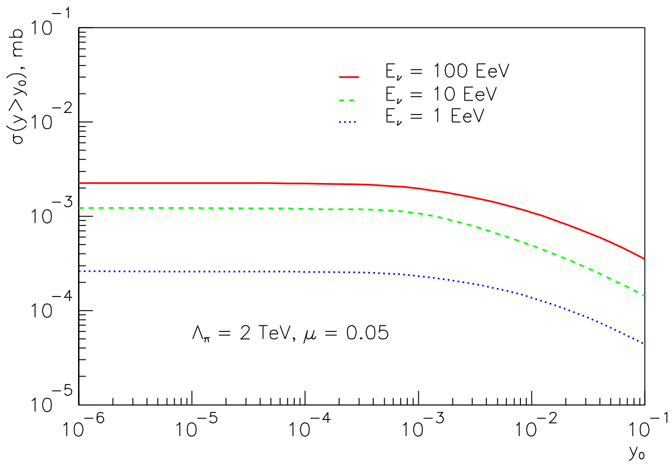

Fig. 7. The inelastic neutrino–proton cross section obtained by integrating the differential cross section in the region $y_0 \leqslant y \leqslant 1$ as a function of y_0, the minimal fraction of the neutrino energy deposited to the proton

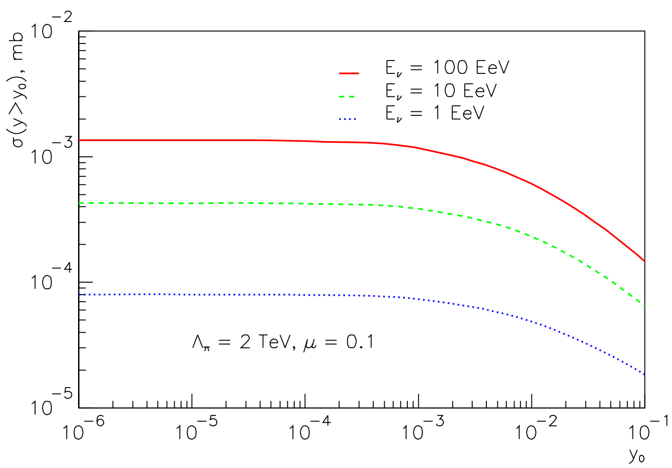

Fig. 8. The same as in Fig. 7 but for the different value of $\mu = 0.1$

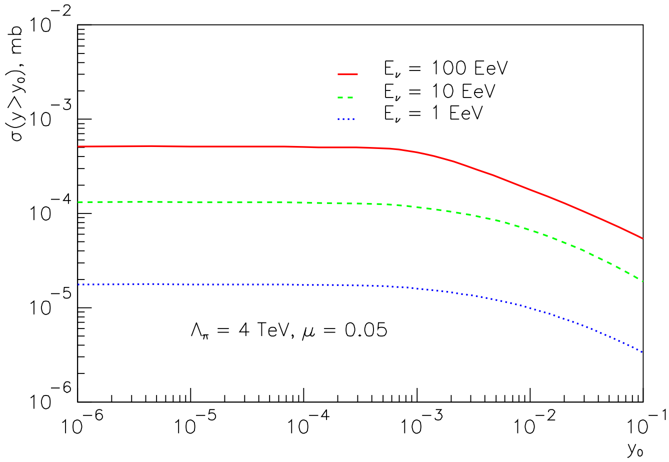

Fig. 9. The same as in Fig. 7 but for the different value of $\Lambda_\pi = 4\,\text{TeV}$

$\mu = 0.05$. Next, Figs. 8 and 9 show the dependence of $\sigma_{\text{el}}(y > y_0)$ on the parameters μ and Λ_π.

Ultra-high energy cosmic neutrinos have not yet been detected (see the non-observation of neutrino-induced events reported by the Fly's Eye [17], the AGASA [18] and the RICE [19] collaborations). A number of experiments under construction will allow one to measure fluxes of such neutrinos within the next few years. Among them are the Pierre Auger Observatory, the IceCube neutrino telescope at the South Pole, the Anita radio detector for balloon flights around the South Pole, as well as the EUSO, SalSA and OWL proposals. We will consider the first possibility [20].

The number of horizontal hadronic air showers with the energy E_{sh} larger than the threshold energy E_{th}, initiated by neutrino–nucleon interactions, is given by

$$N_{\text{sh}} = T N_{\text{A}} \int\limits_{E_{\text{th}}}^{E_{\max}} \mathrm{d}E$$

$$\times \left[\sum_{i=e,\,\mu,\,\tau} \Phi_{\nu_i}(E) \int\limits_0^1 \mathrm{d}y \, \frac{\mathrm{d}\sigma_{\nu N}^{\text{grav}}(E)}{\mathrm{d}y} \, \mathcal{A}(yE)\, \theta(yE - E_{\text{th}}) \right.$$

$$\left. + \sum_{i=e,\,\mu,\,\tau} \Phi_{\nu_i}(E)\, \sigma_{\nu_i N}^{\text{SM}}(E)\, \mathcal{A}(\bar{y}_i E_{\nu_i})\, \theta(\bar{y}_i E - E_{\text{th}}) \right], \quad (30)$$

where $N_{\text{A}} = 6.022 \cdot 10^{23}\,\text{g}^{-1}$, T is the time interval (one year, in our case), and $\mathcal{A}(E)$ is the detector acceptance as a function of the shower energy (in units of km^3 steradian water equivalent $= 10^{15}$ g). The quantity $\Phi_{\nu_i}(E)$ in (30) is the flux of the neutrino of type i. Both neutrino and antineutrino are assumed in the sums in (30). The product $E\,\Phi_{\nu_i}(E)$ is in units of cm^{-2} yr^{-1}. We have taken into account that the energy of the shower resulting from the gravitational interaction is equal to yE, and that this interaction is universal for all types of neutrinos.

For the energy distribution of the neutrino in the SM processes, we have used the approximation

$\mathrm{d}\sigma^{\mathrm{SM}}_{\nu_i N}(y,E)/\mathrm{d}y \simeq \sigma(E)\,\delta(y-\bar{y}_i)$. The inelasticity $\bar{y}_i$ defines the mean fraction of the neutrino energy deposited into the shower in the corresponding SM process. We have put $\bar{y}_e = 1$ for SM *charged* current interactions initiated by the electronic neutrino, while for SM *neutral* interactions initiated by ν_e and for ν_μ/ν_τ events we have taken $\bar{y}_e = \bar{y}_\mu = \bar{y}_\tau = 0.2$ [21].

The number of extensive quasi-horizontal showers induced by so-called cosmogenic neutrinos which can be detected by the array of the southern site of the Pierre Auger Observatory, is presented in Table 1 for several sets of the RS parameters. These values of parameters are chosen in a way not violating the experimental and theoretical bounds presented in Fig. 1.[4] The cosmogenic neutrino flux is taken from [22], assuming $E_{\max} = 3\cdot 10^{21}$ eV. The acceptance of the Auger detector is taken from [23] (it is not assumed that the shower axis falls certainly in the array). The threshold energy E_{th} is chosen to be 10^{17} eV.

For comparison, the SM background is presented in the last row of the Table 1. This value is in agreement with the number obtained recently for the same neutrino flux in [24].

The cosmogenic neutrino flux is the most reliable one, since it relies only on two assumptions:
(i) the observed extremely-high energy cosmic rays contain protons;
(ii) these cosmic rays are primarily extragalactic in origin.
Note, however, that the cosmogenic neutrino flux may be significantly depleted, if a substantial fraction of the cosmic ray primaries are heavy nuclei rather than protons [25].

The cosmogenic neutrino flux not only represents a lower limit on the flux of ultra-high energy neutrinos, but it also can be used to put an upper limit on the neutrino flux. In [26] an upper limit (called the WB bound) on the flux of neutrinos from compact sources which are optically thin to $p\gamma$ and pp interactions (such as active galactic nuclei) has been obtained. The number of showers which can be registered by the Auger detector for this case is shown in Table 2. We have chosen the same threshold energy $E_{\mathrm{th}} = 10^{17}$ eV and put $E_{\max} = 10^{21}$ eV.

Note that the so-called cascade upper limit on transparent neutrino sources [27] (MPR bound) is 43 times higher than the WB bound. It exploits the EGRET data on the diffuse gamma-ray background [28].

The lower bound on the cosmogenic neutrino flux was also obtained under the assumption that the observed extremely-high energy cosmic rays below 10^{20} eV are protons from uniformly distributed extragalactic sources [29]. It uses the fact that the protons are accumulated around the energy $E_{\mathrm{GZK}} = 4\cdot 10^{19}$ eV due to the GZK mechanism [30]. The lower cosmogenic neutrino spectrum is practically cut at $E_\nu \simeq 2\cdot 10^{19}$ eV [29]. Other recent estimates of the cosmogenic neutrino fluxes can be found in [31, 24].

[4] Remember that in our notation $\kappa/\bar{M}_{\mathrm{Pl}} \equiv \mu$, $m_1 \simeq 3.83\,\mu\Lambda_\pi$.

4 Conclusions and discussions

In the present paper we have calculated the contribution from the massive graviton modes to the eikonal in the RS model. The results were applied to the neutrino–nucleon scattering at trans-Planckian energies. Both differential and total cross sections are estimated for the different sets of the parameters of the model. By using differential cross sections, we have calculated the number of quasi-horizontal neutrino-induced air showers which can be detected at the Auger Observatory per year. The estimates were obtained for two fluxes of cosmogenic neutrinos.

The differential cross section, $\mathrm{d}\sigma(y)/\mathrm{d}y$, where y is the fraction of the neutrino energy E_ν deposited to the shower, can reach tens of mb at $y = 0$, depending on the energy (Fig. 6). However, the differential cross section exhibits a rapid fall-off in y, starting at some small y. As a result, the gravitational cross section appears to be approximately one order of magnitude larger than the SM cross section at the same energy. To illustrate this statement, let us fix the parameters of the RS model to be $\Lambda_\pi = 2$ TeV, $\mu = 0.1$. Then we have $(\mathrm{d}\sigma(y)/\mathrm{d}y)|_{y=0} \simeq 4$ mb for $E_\nu = 10^{10}$ GeV (see Fig. 6). As one can see in Fig. 6, $\mathrm{d}\sigma(y)/\mathrm{d}y$ begins to fall rapidly at $y > 10^{-5}$. The numerical calculations show that $\sigma \simeq 4\cdot 10^{-4}$ mb for this case (dashed curve in Fig. 8).

The energy of the neutrino-induced air shower, $E_{\mathrm{sh}} = yE_\nu$, is bounded from below by the threshold energy E_{th}. Thus, the fraction y should obey the inequality $y \geqslant E_{\mathrm{th}}/E_{\max}$, where $E_{\max}$ is the maximum energy in the neutrino spectrum. For $E_{\mathrm{th}} = 10^8$ GeV and $E_{\max} = 10^{11(12)}$ GeV, we get $y \geqslant 10^{-3(4)}$. Thus, the air shower event rate is defined by the region of y in which the neutrino–nucleon cross section $\mathrm{d}\sigma(y)/\mathrm{d}y$ is significantly reduced in comparison with its magnitude at $y = 0$. Nevertheless, the gravity contribution to the event rate at the Auger detector is several times larger than the SM background, as one can see from Tables 1 and 2.

Recently, model independent bounds on the inelastic neutrino–nucleon cross section derived from the AGASA [18] and RICE [19] search results on neutrino events were obtained [24]. The bounds exploit the cosmogenic neutrino fluxes from [22, 29]. However, they were derived under the assumption that the *total* neutrino energy goes into the shower energy; that is, $y = 1$. As we have seen, this is not the case for the gravitational interactions originating from t-channel KK gravitons, which prefer $y \ll 1$.[5] Generally, in order to extract an upper limit on σ_{tot}, the dependence of $\mathrm{d}\sigma(y)/\mathrm{d}y$ on y is needed. So we conclude that the bounds from [24] cannot be directly apply to the neutrino–nucleon cross sections derived in our scheme.

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[5] In processes initiated by graviton t-channel exchanges in large extra dimensions the mean energy loss is also small, as was pointed out in [34]. On the contrary, in the process of black hole production, the neutrino loses most of its initial energy ($y \approx 1$).

Table 1. Yearly event rates for nearly horizontal neutrino-induced showers with $\theta_{\mathrm{zenith}} > 70^\circ$ for the cosmogenic neutrino flux from [22] for three sets of the parameters. The number of events corresponds to one side of the Auger ground array

	$\Lambda_\pi = 2\,\mathrm{TeV}$, $\mu = 0.10$	$\Lambda_\pi = 3\,\mathrm{TeV}$, $\mu = 0.05$	$\Lambda_\pi = 3\,\mathrm{TeV}$, $\mu = 0.10$
SM+grav	0.81	0.66	0.43
SM		0.24	

Table 2. The same as in Table 1 but for the Waxman–Bahcall neutrino flux [26]

	$\Lambda_\pi = 2\,\mathrm{TeV}$, $\mu = 0.10$	$\Lambda_\pi = 3\,\mathrm{TeV}$, $\mu = 0.05$	$\Lambda_\pi = 3\,\mathrm{TeV}$, $\mu = 0.10$
SM+grav	1.03	0.78	0.49
SM		0.28	

Appendix

In this appendix we calculate the dependence of the eikonal (20) on the variable u (19). Let us define

$$I(u) = \int_0^\infty \mathrm{d}x \, J_0(ux) \, R(x), \tag{A.1}$$

with

$$R(x) = \frac{I_2(x)}{I_1(x)} \tag{A.2}$$

being the ratio of two modified Bessel functions.

It easily to see from (A.1) that $I(u) \to u^{-1}$ at $u \to 0$, since $R(x) \to 1$ at $x \to \infty$. The asymptotics of $I(u)$ at large u is defined by the behavior of the integrand at small x which looks like

$$R(x) \simeq \frac{x}{4} - \frac{x^3}{96} + \frac{x^5}{1536} - \frac{x^7}{23040} + O(x^9). \tag{A.3}$$

Let us now demonstrate that at $u \to \infty$ the function $I(u)$ (A.1) decreases faster that any fixed power of u^{-1}. By using the well-known relation [32]

$$x^{\nu-1} J_{\nu-1}(ux) = \frac{1}{u} \left(\frac{\mathrm{d}}{x \, \mathrm{d}x} \right) \left[x^\nu J_\nu(ux) \right], \tag{A.4}$$

and integrating (A.1) by parts k times, we obtain

$$I(u) = \frac{1}{u^k} \int_0^\infty \mathrm{d}x \, J_k(ux) F_k(x), \tag{A.5}$$

where $J_k(z)$ is the Bessel function, and

$$F_k(x) = (-1)^k \, x^{k+1} \left(\frac{\mathrm{d}}{x \, \mathrm{d}x} \right)^k \left[\frac{R(x)}{x} \right]. \tag{A.6}$$

The function $F_k(x)$ (A.6) has the following properties: it is proportional to x^{k+1} at $x \to 0$, and it decreases as x^{-k} at $x \to \infty$. For any positive integer k, it depends only on x and on the ratio $R(x)$, see (A.2), (but not on $I_1(x)$ and $I_2(x)$ separately) due to the following relations between modified Bessel functions [32]:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x} I_1(x) &= I_2(x) + \frac{1}{x} \, I_1(x), \\ \frac{\mathrm{d}}{\mathrm{d}x} I_2(x) &= I_1(x) - \frac{2}{x} \, I_2(x). \end{aligned} \tag{A.7}$$

For instance, for $k = 1$ one has

$$F_1(x) = -1 + \frac{4R(x)}{x} + \left[R(x) \right]^2. \tag{A.8}$$

This expression has asymptotics $x^2/48$ and x^{-1} at small and large x, respectively. For $k = 2$ one gets

$$\begin{aligned} F_2(x) = \frac{1}{x} \Big\{ &-6 \left[1 - \frac{4R(x)}{x} \right] - 2x \, R(x) + 12 \left[R(x) \right]^2 \\ &+ 2x \left[R(x) \right]^3 \Big\}. \end{aligned} \tag{A.9}$$

The asymptotics of $F_2(x)$ are $x^3/192$ and $3\,x^{-2}$.

Since k is an arbitrary positive integer, we conclude from (A.5) and (A.6) that $\lim\limits_{u \to \infty} u^a \, I(u) = 0$ for any $a > 0$.

The integral in (A.5), contrary to the original one in (A.1), converges rapidly at $x \to \infty$ for $k \geqslant 2$, and could be used for numerical calculations. It cannot be calculated analytically. However, there exists an expression which approximates our integral with a very high accuracy:

$$\bar{I}(u) = \int_0^\infty \mathrm{d}x \, J_0(ux) \, \bar{R}(x), \tag{A.10}$$

with

$$\bar{R}(x) = \frac{\sqrt{3}}{2} \, \frac{x}{\sqrt{x^2 + 12}}. \tag{A.11}$$

The function $\bar{R}(x)$ has the following expansion at $x^2 < 12$ (compare with (A.3)):

$$\bar{R}(x) \simeq \frac{x}{4} - \frac{x^3}{96} + \frac{x^5}{1536} - \frac{5\,x^7}{110592} + O(x^9). \tag{A.12}$$

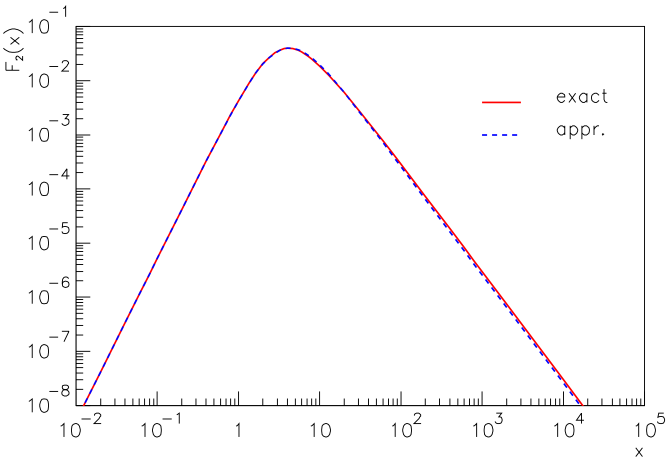

Fig. 10. The exact integrand versus the approximate one (after the integration of both integrals by parts twice). See the appendix for details

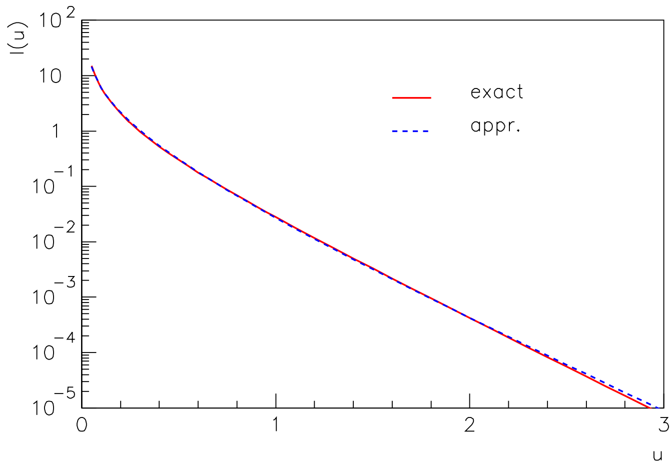

Fig. 11. The input integral versus the approximate one as a function of the dimensionless variable u (19). See the appendix for details

The integral (A.10) can be found in a Table [33]:

$$\bar{I}(u) = \frac{\sqrt{3}}{2u} \exp(-2\sqrt{3}\, u). \tag{A.13}$$

We can integrate the RHS of (A.10) twice by parts,

$$\bar{I}(u) = \frac{1}{u^2} \int_0^\infty \mathrm{d}x\, J_2(ux)\, \bar{F}_2(x), \tag{A.14}$$

and compare $\bar{F}_2(x) = (3\sqrt{3}/2)\, x^3/(x^2+12)^{5/2}$ with the corresponding function $F_2(x)$; see (A.9). The result of our calculations is presented in Fig. 10. Equation (A.13) gives practically the same dependence on variable u as the numerical integration of the exact expression by using (A.5) (with $k = 2$) does; see Fig. 11.[6] Thus, $I(u)$ exhibits an exponential fall-off (as we expected; see above), and it becomes as small as $I(u) \simeq 0.01$ already at $u = 1.2$.

Taking all that was said above into account, we put $I(u) \to \bar{I}(u)$, which results in the analytical expression for the eikonal as presented in the text, (21).

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[6] Some disagreement between the two curves at $u \gtrsim 2.5$ is not important, since $I(u)$ (and, consequently, $\chi_{\rm mass}(s,u)$) is strongly suppressed in this region. Note that the region of very small u also gives a negligible contribution to the eikonal amplitude (15).

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